Spreading of damage in the Domany-Kinzel cellular automaton: a mean-field approach

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Abstract

We present a detailed analytical formulation and a mean-field approximation analysis of the spreading of damage in the Domany-Kinzel cellular automaton. Our results show that the system exhibits a chaotic state besides the frozen and active. These results are in agreement with recent numerical simulations. Also we study the conjugate fields associated to the order parameters of the active and chaotic phases.

1. Introduction

It is known that the Domany-Kinzel cellular automaton [1] exhibits an active state besides a frozen state devoided of particles. The frozen state is an absorbing or trapping state from which the system cannot escape once it is reached. The active state is characterized by having particles that are continuously been created and annihilated with a nonzero density of particles. Recently [2], it has been found by numerical simulation that, in the active state, the automaton can behave in two different ways when one looks to the spreading of a damage. If the damage spreads throughout the system then the automaton is sensitive to the initial damage and the system is said to be in a chaotic state. Otherwise, it is in an active non-chaotic state, and there is no spreading of damage.

The numerical simulation of the spreading of damage [2] is performed by using the technique of creating a replica of the system with an "initial damage" [2]. The system and its replica are then imposed to evolve in time subjected to the same noise which means to use the same random number sequence for both replicas. Here we present an analytical formulation of the spreading of damage. Two replicas of the automaton are coupled together in such a way that they are subject to the same noise but do not loose their individuality. The phase diagram of the coupled system is obtained by using
mean field approximations up to the order of two site correlation. Finally, we study the conjugate fields associated to active state and to the chaotic state and calculate the susceptibilities associated to these fields. As expected, the susceptibilities diverges at the corresponding critical lines.

2. Domany-Kinzel cellular automaton

The system studied is a stochastic one-dimensional lattice, with \( L \) sites that evolves in time according to local rules given by the Domany-Kinzel cellular automaton [1]. The configurations of the system are described in terms of a set of occupation variables \( \{ \sigma_i \} \), with \( \sigma_i = 1 \) or \( 0 \) according whether the site \( i \) is occupied or empty. The system evolves at discrete time steps and all the sites are updated simultaneously.

Let \( P_t(\sigma) \) be the probability of state \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_L) \) at time \( t \). The time evolution of \( P_t(\sigma) \) is governed by the equation

\[
P_{t+1}(\sigma) = \sum_{\sigma'} W(\sigma \mid \sigma') P_t(\sigma'),
\]

where \( W(\sigma \mid \sigma') \) is the transition probability from state \( \sigma' \) to state \( \sigma \), and must obey the condition

\[
\sum_{\sigma} W(\sigma \mid \sigma') = 1,
\]

for any state \( \sigma' \). Since we are leading with a cellular automaton, which means that all sites are updated simultaneously, \( W(\sigma \mid \sigma') \) must be written as

\[
W(\sigma \mid \sigma') = \prod_{i} w_i(\sigma_i \mid \sigma').
\]

where the quantity \( w_i(\sigma_i \mid \sigma') \) is the probability that site \( i \), at time \( t + 1 \), has the value \( \sigma_i \) given that, at time \( t \), the state of the system is \( \sigma' \). It has the property

\[
w_i(0 \mid \sigma') + w_i(1 \mid \sigma') = 1,
\]

so that Eq. (2) is satisfied. For the Domany-Kinzel cellular automaton, \( w_i(\sigma_i \mid \sigma') \) will be of the form \( w_i(\sigma_i \mid \sigma') = w_{DK}(\sigma_i \mid \sigma'_{i-1}, \sigma'_{i+1}) \), and is given by the following rules:

\[
\begin{array}{cccccc}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & p_2 & p_1 & p_1 & 0 & 0 \\
0 & 1 - p_2 & 1 - p_1 & 1 - p_1 & 1
\end{array}
\]

This model exhibits an stable [1–3] absorbing state devoided of particles, i.e. a state where \( \sigma_i = 0 \) for any site in the lattice. In the recent years a great variety of models with absorbing states have been studied [4,5]. The universality classes of these non-equilibrium systems as other aspects of the stochastic dynamics of these models have been extensively analysed. Some models evolve continuously in time.
(governed by a master equation) and others, such as the one considered here, evolve via a discrete dynamics (cellular automata). In general, models with absorbing states exhibit a dynamic phase transition from an active state to the absorbing state with the same universal critical behavior.

In order to find an expression for the mean value of occupied sites \( x \), the order parameter of the active state, we will make some definitions, and set up some evolution equations. Let us denote by \( \langle f(\sigma) \rangle_\ell \) the mean value of the state function \( f(\sigma) \) at time \( \ell \), given by

\[
\langle f(\sigma) \rangle_\ell = \sum_\sigma f(\sigma)P_\ell(\sigma)
\]

We will consider the case where the system is homogeneous. According to Eq. (1) and using Eq. (6), the time evolution for the mean occupation number \( x_\ell = P_\ell(1) \) is given by

\[
P_{\ell+1}(1) = \langle w_{DK}(1 \mid \sigma_{i-1}, \sigma_i) \rangle_\ell.
\]

or, using the rules in (5),

\[
P_{\ell+1}(1) = 2p_1P_\ell(10) + p_2P_\ell(11).
\]

Similarly, the evolution equation for \( P_\ell(11) \) is given by

\[
P_{\ell+1}(11) = \langle w_{DK}(1 \mid \sigma_{i-1}, \sigma_i)w_{DK}(1 \mid \sigma_i\sigma_{i+1}) \rangle_\ell.
\]

or, using the rules in (5), by

\[
P_{\ell+1}(11) = p_1^2[P_\ell(101) + P_\ell(010)] + 2p_1p_2P_\ell(110) + p_2^2P_\ell(111).
\]

The equation for \( P_\ell(10) \) is not necessary because \( P_\ell(10) = P_\ell(1) - P_\ell(11) \). Equations for the evolution of other correlations can also be written. In Section 4 we will make an analytical study of the frozen-active transition by using the above equations.

3. Joint evolution

Martins et al. [2] analysed the effect of damage spreading in the active state. They have found, through numerical simulations, that the active state can have chaotic nature for given values of parameters \( p_1 \) and \( p_2 \). They used a technique of creating a replica of the system with an "initial damage". The system and its replica are then imposed to evolve in time subjected to the same noise. If a spreading of damage occurs then the system is sensitive to the initial conditions and this characterizes an active state with chaotic nature. To study this feature another order parameter, besides \( x \), must be introduced and this is the Hamming distance \( \psi \). To make the analytical study of the evolution in time of \( \psi \) we must construct the equation for the joint evolution of the two system subjected to the same noise and stabilize all the probabilities concerning this evolution.
Let us call \( \{ \sigma_i \} \) the set of site occupation variables of the system, as before, and \( \{ \tau_i \} \) the set of site occupation variables of the replica system. The Hamming distance is defined by

\[
\psi_\ell = \langle (\sigma_i - \tau_i)^2 \rangle_\ell.
\]  (11)

Let \( P_\ell(\sigma ; \tau) \) be the probability that at time \( \ell \) the system is in state \( \sigma \) and the replica is in state \( \tau \). The evolution equation for the joint probability is given by

\[
P_{\ell+1}(\sigma; \tau) = \sum_{\sigma', \tau'} W(\sigma; \tau | \sigma', \tau') P_\ell(\sigma'; \tau'),
\]  (12)

where

\[
W(\sigma; \tau | \sigma', \tau') = \prod_i w(\sigma_i; \tau_i | \sigma_{i-1}', \sigma'_i; \tau_{i-1}', \tau'_i)
\]  (13)

is the joint transition probability of the two systems (subjected to the same noise) of going from the joint state \((\sigma'; \tau')\) to the joint state \((\sigma ; \tau)\). The transition probabilities \(w(\sigma_i; \tau_i | \sigma_{i-1}', \sigma'_i; \tau_{i-1}', \tau'_i)\) obey the following conditions

\[
\sum_{\sigma_i} \sum_{\sigma_{i-1}'} \sum_{\tau_i} \sum_{\tau_{i-1}'} w(\sigma_i; \tau_i | \sigma_{i-1}', \sigma'_i; \tau_{i-1}', \tau'_i) = 1,
\]  (14)

\[
\sum_{\sigma_{i-1}'} \sum_{\tau_{i-1}'} w(\sigma_i; \tau_i | \sigma_{i-1}', \sigma'_i; \tau_{i-1}', \tau'_i) = w_{DK}(\sigma_i | \sigma_{i-1}', \sigma'_i),
\]  (15)

and

\[
\sum_{\sigma_{i-1}'} \sum_{\tau_{i-1}'} w(\sigma_i; \tau_i | \sigma_{i-1}', \sigma'_i; \tau_{i-1}', \tau'_i) = w_{DK}(\tau_i | \tau_{i-1}', \tau'_i)
\]  (16)

where \(w_{DK}(\sigma_i | \sigma_{i-1}', \sigma'_i)\) are given in (5). The construction of the joint probabilities must obey Eqs. (14), (15) and (16) and they are given below:

\[
\begin{array}{ccccccccc}
11 & 11 & 10 & 10 & 00 & 00 & 11 & 00 & 10 & 00 & 11, 10 \\
10 & p_2 & p_1 & 0 & 0 & 0 & a & \\
10 & 0 & 0 & 0 & p_2 & p_1 & b & \\
01 & 0 & 0 & 0 & 0 & c & \\
00 & 1 - p_2 & 1 - p_1 & 1 - p_2 & 1 - p_1 & d &
\end{array}
\]  (17)

The other transition probabilities are obtained using the transformations: (a) \( \sigma_i' \leftrightarrow \sigma_{i-1}' \), (b) \( \tau_i' \leftrightarrow \tau_{i-1}' \) or (c) \( \sigma_i \leftrightarrow \tau_i \), \( \sigma_i' \leftrightarrow \tau_i' \) and \( \sigma_{i-1}' \leftrightarrow \tau_{i-1}' \) and properties given in Eqs. (14), (15) and (16).

We observe that \( a + b = p_2, c + d = 1 - p_2, a + c = p_1 \) and \( b + d = 1 - p_1 \). Therefore, if \( a \) is fixed we obtain \( b, c \) and \( d \) by \( b = p_2 - a, c = p_1 - a \) and \( d = 1 - p_1 - p_2 + a \).

The most important point in the evolution of the two systems subjected to the same noise is focused in the intrinsic prerrogatives that must be obeyed by the parameters \( a, b, c \) and \( d \). The choice of these parameters, that is, the dependence of them on \( p_1 \)
and \( p_2 \), must be such that the time evolutions of the systems are subjected to the same noise. This is accomplished if we impose the probability \( a \) to be

\[
a = \min\{p_1, p_2\}. \tag{18}
\]

If \( p_1 \leq p_2 \) then \( a = p_1, b = p_2 - p_1, c = 0 \) and \( d = 1 - p_2 \). If \( p_1 \geq p_2 \) then \( a = p_2, b = 0, c = p_1 - p_2 \) and \( d = 1 - p_1 \).

Suppose that one is doing a numerical simulation for the case \( p_1 < p_2 \) and that in a certain time step the configuration is one in which \( \sigma_{i-1} = 1, \sigma_{i+1} = 1, \tau_{i-1} = 1, \) and \( \tau_{i+1} = 0 \). In the next time step the values of \( \sigma_i \) and \( \tau_i \) are obtained by generating a random number and considering the three possibilities: (i) the random number is less than \( p_1 \), (ii) between \( p_1 \) and \( p_2 \), and (iii) greater than \( p_2 \). According to the Domany-Kinzel rules one should have \( \sigma_i = 1 \) in cases (i) and (ii), and \( \sigma_i = 0 \) in case (iii). For the other system one should have \( \tau_i = 1 \) in case (i) and \( \tau_i = 0 \) in cases (ii) and (iii). Therefore, the possibilities for \( (\sigma_i, \tau_i) \) are: 1, 1 for case (i), 1, 0 for case (ii) and 0, 0 for case (iii), and 0, 1 is excluded. This is in accordance with the choice 18 which gives, for this case, \( c = 0 \).

A distinct approach concerning the study of spreading of damage in the Domany-Kinzel cellular automaton was the one introduced by Kohring and Schreckenberg [31]. Their scheme corresponds to choose the parameters \( a, b, c \) and \( d \) such that \( a = p_1 p_2 \), so that \( b = p_2 (1 - p_1) \), \( c = p_1 (1 - p_2) \) and \( d = (1 - p_1) (1 - p_2) \). Therefore, in their approach, it can be necessary to use two random numbers to update a given site when the states are different in both systems, as in the example of the preceding paragraph. Indeed if \( a = p_1 p_2 \) it is necessary to generate independently two random numbers: one for \( p_1 \) and other for \( p_2 \).

Now, we are in position to set up the equation for the evolution of the order parameter \( \psi \) associated to the chaotic phase. Using Eq. (11) and the condition \( \sum_\sigma P_\ell(\sigma; \tau) = P_\ell(\tau) \) the Hamming distance is written as

\[
\psi_\ell = P_\ell(1; 0) = P_\ell(0; 1). \tag{19}
\]

Using (12) and (19) we obtain

\[
P_{\ell+1}(1; 0) = \langle w(1; 0 \mid \sigma_i, \sigma_{i+1}; \tau_i, \tau_{i+1}) \rangle_\ell. \tag{20}
\]

From this expression and using the rules in (17) we get the following expressions for the time evolution of \( \psi_\ell \)

\[
P_{\ell+1}(1; 0) = p_2 P_\ell(11; 00) + 2 p_1 P_\ell(10; 00) + 2(b + c) P_\ell(11; 10). \tag{21}
\]

We can see that this equation involves probabilities of pairs of sites of each system. The time evolution for these quantities can be deduced using Eq. (12) and the rules given in (5) and (17). They are formally given by

\[
P_\ell(11; 00) = \langle w(10 \mid \sigma_{i-1}, \sigma_i; \tau_{i-1}, \tau_i) w(10 \mid \sigma_i, \sigma_{i+1}; \tau_i, \tau_{i+1}) \rangle_\ell, \tag{22}
\]

\[
P_\ell(10; 00) = \langle w(10 \mid \sigma_{i-1}, \sigma_i; \tau_{i-1}, \tau_i) w(00 \mid \sigma_i, \sigma_{i+1}; \tau_i, \tau_{i+1}) \rangle_\ell, \tag{23}
\]
Using the rules given in (17), the right-hand sides of the above equations can be written in terms of the correlations \( P_\ell(11;11) \), \( P_\ell(11;10) \), etc. Since they are too cumbersome we will not write it down.

4. Dynamic mean-field approximation

Equations for the time evolution of the order parameters \( x_\ell = P_\ell(1) \) and \( \psi_\ell = P_\ell(1;0) \) depend on higher order correlations functions as can be seen in Eqs. (8) and (21). These variables have evolution equations that involve correlations of clusters of three particles and so on. So we have an infinite chain of coupled equations. The starting point to analyse the time evolution of order parameters, and determine the phase diagram, is a truncation of this chain. This will be the subject of this section.

We truncate the equations for the time evolution of the correlations using a dynamical mean-field approximation [6].

4.1. One-site approximation

The simplest mean-field approximation is the one-site approximation where the cluster probabilities are decoupled as a product of the probabilities of each site, that is,

\[
P_\ell(\sigma_{i-1}, \sigma_i) = P_\ell(\sigma_{i-1})P_\ell(\sigma_i)
\]

and

\[
P_\ell(\sigma_{i-1}, \sigma_i; \tau_{i-1}, \tau_i) = P_\ell(\sigma_{i-1}; \tau_{i-1})P_\ell(\sigma_i; \tau_i)
\]

So, in the context of this approach, Eqs. (8) and (21) are then written as

\[
x_{\ell+1} = p_2 x_\ell^2 + 2p_1 x_\ell (1 - x_\ell),
\]

and

\[
\psi_{\ell+1} = p_2 \psi_\ell^2 + 2p_1 \psi_\ell (1 - x_\ell - \psi_\ell) + 2(b + c)(x_\ell - \psi_\ell)\psi_\ell.
\]

where we have used the equalities \( P_\ell(1;1) = P_\ell(1) - P_\ell(1;0) \) and \( P_\ell(0;0) = 1 - P_\ell(1) - P_\ell(1;0) \).

The trivial fixed point of Eqs. (27) and (28), \( x = 0, \psi = 0 \), corresponds to a vacuum absorbing state (frozen state). Another fixed point is the one where \( x = (2p_1 - 1)/(2p_1 - p_2) \) and \( \psi = 0 \) that corresponds to an active state. The transition line from the absorbing state to the active state is given by \( p_1 = 1/2 \).

The third possibility to be considered is the one where \( \psi \neq 0 \) and \( x \neq 0 \). Expanding Eq. (28) up to linear terms in \( \psi \) we find that the transition line from the non-chaotic active phase \( (x \neq 0 \text{ and } \psi = 0) \) to a chaotic active phase \( (x \neq 0 \text{ and } \psi \neq 0) \) is given by \( p_2 = 2p_1/3 \).
The phase diagram in this case is given in Fig. 1. We can see that the one-site approximation has a qualitative agreement with the phase diagram obtained through numerical simulations [2]. The only difference is that in the context of this approach, the chaotic-active transition line meets the frozen-active line in a point where \( p_2 \neq 0 \), whereas in the simulations they meet at \( p_2 = 0 \). However, as we will see below, by increasing the order of truncation, that is, when more correlations are taken into account, the meeting point approaches the line \( p_2 = 0 \).

4.2. Pair approximation

In the dynamical mean field at the level of pair approach the probability of clusters of three sites are approximated by [6]

\[
P(\sigma_{i-1}, \sigma_i, \sigma_{i+1}) = \frac{P(\sigma_{i-1}, \sigma_i)P(\sigma_i, \sigma_{i+1})}{P(\sigma_i)}
\]  

(29)

For the case of the joint probabilities, which appear in the present problem as probabilities involving variables of the two systems, the probability of a cluster of three sites in system \( \{\sigma\} \) and a cluster of three sites in system \( \{\tau\} \) is approximated by

\[
P(\sigma_{i-1}, \sigma_i, \sigma_{i+1}; \tau_{i-1}, \tau_i, \tau_{i+1}) = \frac{P(\sigma_{i-1}, \sigma_i; \tau_{i-1}, \tau_i)P(\sigma_i, \sigma_{i+1}; \tau_i, \tau_{i+1})}{P(\sigma_i; \tau_i)}
\]  

(30)

In this approximation equations for \( x_{t+1} \) and \( \psi_{t+1} \) given respectively in (8) and (21) retain pair correlations. There are six independent variables that are chosen to be: \( x_t = P_t(1), P_t(11), \psi_t = P_t(1;0), P_t(11;10), P_t(11;00) \) and \( P_t(10;00) \). The other two-site and one-site variables can be written in terms of this set. So in this case Eqs. (8), (10), (21), (22), (23) and (24) become a closed set of equations for these six variables.
The frozen state corresponds to the trivial solution of these equations. The second type of solution that may appear is the active state, described by $x \neq 0, \psi = 0$. It occurs for high values of $p_1$ and $p_2$ and the critical line separating the frozen state to the active state is given by $p_2 = 3p_1(2/3 - p_1)/(1 - p_1)^2$. Decreasing $p_2$, the active state becomes unstable giving rise to a new stationary state. This new state is identified by having a chaotic nature where $\psi$, the Hamming distance, is different from zero. The phase diagram shown in Fig. 2 displays the regions of occurrence of each type of stationary state. This phase diagram approaches better the phase diagram obtained by simulations [2]. But the transition line active-chaotic still meets the line active-frozen in a point where $p_2 \neq 0$. We believe that the lines could meet at $p_2 = 0$ if more correlations are taken into account in the mean-field approximation analysis.

Kohring and Schreckenberg [3] also studied the spreading of damage by a mean-field approximation. In the phase diagram obtained by them the two lines active-chaotic and active-frozen meet at $p_2 = 0$. But they found a reentrant behavior which is not present in simulations. The different results obtained by the present approach and theirs may be due to the use of distinct choices of parameter $a$ and also distinct mean-field approximations.

5. Conjugate fields

In equilibrium statistical mechanics, a conjugate field $H$ associated to a quantity $M$ is introduced by adding, in the Hamiltonian that defines the system, a linear term in $H$. In nonequilibrium systems this cannot be done because the system is defined by a set of dynamic rules and not by a Hamiltonian. A possible generalization of the concept of conjugate field for nonequilibrium system can however be done by adding into the transition rates a linear term in $H$. As we will see below this leads to the following
properties [7,8]: (a) when \( H \) is nonzero, \( M \) is also nonzero, and (b) the generalized susceptibility \( \chi = \partial M / \partial H \) diverges at critical point.

5.1. Conjugate field associated to the density

In this subsection we will consider that the system is allowed to get out of the vacuum state with a probability \( w(1|0,0) = p_0 \). The conjugate field associated to the density \( x = P(1) \) is then \( p_0 \). In this case the time evolution equation for the density of occupied sites will be given by

\[
P_{\ell+1}(1) = p_2 P_{\ell}(11) + 2p_1 P_{\ell}(10) + p_0 P_{\ell}(00).
\]

(31)

We will analyse this equation in the one site approximation. In this case the time evolution for \( x_\ell \) will be

\[
x_{\ell+1} = p_2 x_\ell^2 + 2p_1 x_\ell (1 - x_\ell) + p_0 (1 - x_\ell)^2
\]

(32)

In the stationary regime, the relation between the order parameter \( x \) and the conjugate field \( p_0 \) will be given by the equation

\[
p_0 = \frac{(1 - 2p_1)x + (2p_1 - p_2)x^2}{(1 - x)^2}
\]

(33)

In order to see how the fluctuations behave near the transition from the vacuum state to the active state we calculate the generalized susceptibility

\[
\chi_x = \frac{\partial x}{\partial p_0}
\]

(34)

at \( p_0 = 0 \). At the transition, where \( p_1 = 1/2 \), the susceptibility diverges as

\[
\chi_x \sim |2p_1 - 1|^{-1}
\]

(35)

characterizing a second order phase transition. The critical exponent is classical, as expected in a mean-field analysis.

5.2. Conjugate field associated to \( \psi \).

The conjugate field associated to the order parameter \( \psi \) is a quantity \( h \) which weakens the coupling of the two systems that evolves in time according to the same rules and submitted to the same noise. In order to make an analytical study of this concept we will consider the joint probability of evolution of the two systems written as

\[
W_C(\sigma; \tau | \sigma'; \tau') = (1 - h)W(\sigma; \tau | \sigma'; \tau') + hW(\sigma | \sigma')W(\tau | \tau'),
\]

(36)

where \( W(\sigma; \tau | \sigma'; \tau') \) is defined in (13) and \( W(\sigma; \sigma') \) and \( W(\tau; \tau') \) are defined in (3). The quantity \( h \) is interpreted as the probability of decoupling the two systems. We observe that if \( h = 1 \) the two systems are totally decoupled.
In this context and using the one-site mean-field approximation, the time evolution equation for the order parameter $\psi_t$ is given by

$$\psi_{t+1} = 2p_1 \psi_t - (b + c + p_1 - \frac{1}{2}p_2)\psi_t^2 + 2(b + c - p_1)x_t \psi_t + h\{\frac{1}{2}p_2(1 - p_2)(2x_t - \psi_t)^2 + 4p_1(1 - p_1)(2x_t - 2\psi_t - \psi_t + \psi_t^2)\}.$$  \hfill (37)

In the stationary regime, the susceptibility $X_\psi$ is defined as

$$X_{\psi} = \frac{\partial \psi}{\partial h}$$  \hfill (38)

and is calculated at $h = 0$. Near the critical line it behaves as

$$X_\psi \sim |A - 1|^{-1},$$  \hfill (39)

where

$$A = \frac{2(2p_1^2 - 3p_1p_2 + p_2)}{2p_1 - p_2}.$$  \hfill (40)

We can see that when $A \to 1$, what happens over the critical line, that is, when $p_2 = 2p_1/3$, the susceptibility $X_\psi$ diverges. This behavior is consistent with a second order phase transition.

Near the critical line we have also $\psi \sim (A - 1)$ and at the critical line $h \sim \psi^2$. From these results and from (39) we conclude that the critical exponents are $\gamma = 1$, $\beta = 1$, and $\delta = 2$. These are classical exponents, expected in a mean-field analysis.

The behavior of $X_x$ and $X_\psi$ was studied through numerical simulations by Martins et al. [7] and by Tsallis and Martins [8], respectively. The mean-field analysis, here developed, predicts the same type of behavior for the two susceptibilities.

6. Conclusion

We have made an analytical study of the Domany-Kinzel cellular automaton. We constructed the evolution equation for the joint probability of two systems (a system and its replica, which differ by a finite fraction of sites) that obey the Domany-Kinzel cellular automaton rules and evolve in time subjected to the same noise. We have verified, through a dynamical mean-field analysis that the model presents three types of phases: frozen (absorbing), active and chaotic. Our results corroborate the numerical simulations done by Martins et al. [7]. We also studied conjugated fields associated to the order parameters $x$ (the mean occupation number) and $\psi$ (the Hamming distance). This analysis was made using mean-field approximation and are in agreement with the numerical simulations done by Tsallis and Martins [8].
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